FOUNDATIONS OF PHYSICS I (1101/6)

Summary of Second Term Notes

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1 ROTATIONAL MOTION OF RIGID BODIES–Ch.10/11

1.1 Circular Motion of Point Mass

1.1.1 Definitions of Physical Quantities

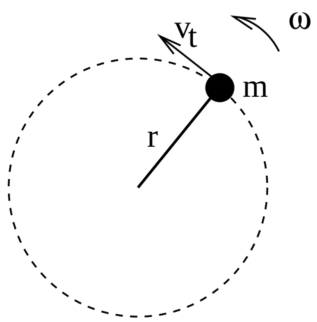

Figure 1: Circular Motion

Angular Velocity:

$$\omega = \frac{d\theta}{dt} = \frac{v_t}{r}$$

Angular Acceleration:

$$\alpha = \frac{d\omega}{dt} = \frac{a_t}{r}$$

Period:

$$T = \frac{2\pi r}{v_t} = \frac{2\pi}{\omega}$$

Centripetal Acceleration:

$$a_c = \frac{v_t^2}{r}$$

Rotational Kinetic Energy:

$$\mathrm{KE}_{\mathrm{rot}} = \frac{1}{2}mv_t^2 = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}I\omega^2$$

Moment of Inertia about a Given Axis of Rotation:

$$I = mr^2$$

Torque

$$\tau = rF_{\perp} = rF\sin(\theta) = I\alpha = \frac{dL}{dt}$$

Angular Momentum

$$L = I\omega = mv_t r$$

NOTE:

- Angular velocity, angular acceleration, torque and angular momentum are **vector** quantities. The above formulas give their magnitudes. Their directions are always perpendicular to the plane of rotation, with the actual direction determined by the **right hand rule:** curl the fingers of your right hand along the direction of rotation, and your thumb points along the direction of the corresponding vector.

- Moment of inertia, torque, angular momentum are always defined relative to a given axis of rotation. Different choices of axis give different values of these quantities for the same mass moving with the same linear velocity.

1.1.2 Torque and Angular Momentum as Cross Products

The **Cross Product**

$$\vec{C} = \vec{A} \times \vec{B}$$

of two vectors $\vec{A}$ and $\vec{B}$ is defined as the vector with magnitude:

$$C = AB\sin(\theta)$$

where θ is the angle subtended by the two vectors. The direction of the cross product $\vec{C}$ is always perpendicular to both $\vec{A}$ and $\vec{B}$ with the direction of the arrow determined by the **right hand rule**: point the fingers of your right hand from $\vec{A}$ to $\vec{B}$, and your thumb points along the direction of the cross product $\vec{A} \times \vec{B}$.

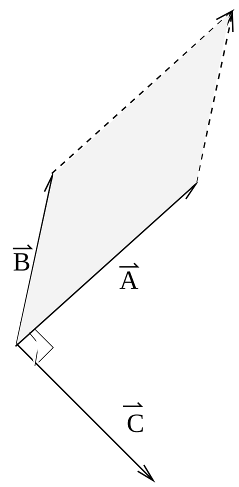

Figure 2: Cross Product

NOTE:

- The magnitude of the cross product has a geometrical interpretation as the area of the parallelogram subtended by the vectors $\vec{A}$ and $\vec{B}$.
- $\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$ (i.e. they have the same magnitude but point in opposite directions.
- If $\vec{A}$ and $\vec{B}$ are parallel, $\theta = 0$ and the cross product is zero.

Torque about an axis O on a point mass due to a force $\vec{F}$:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where $\vec{r}$ is the position vector of the mass relative to the axis of rotation O.

Angular Momentum about an axis O of a point mass m moving with velocity $\vec{v}$:

$$\vec{L} = \vec{r} \times \vec{p}$$

where $\vec{r}$ is the position of the mass relative to the axis and $\vec{p} = m\vec{v}$ is its linear momentum.

1.1.3 Comparison between linear and rotational quantities

Linear Motion	Circular Motion
Velocity: $\vec{v}$	Angular Velocity: $\vec{\omega}$
Inertial Mass: m	Moment of Inertia: I
Kinetic Energy: $\frac{1}{2}mv^2$	Kinetic Energy: $\frac{1}{2}I\omega^2$
Force: $\vec{F}$	Torque: $\vec{\tau} = \vec{r} \times \vec{F}$
Linear Momentum: $\vec{p} = m\vec{v}$	Angular Momentum: $\vec{L} = I\vec{\omega} = \vec{r} \times \vec{p}$
Second Law: $\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$	Second Law: $\vec{\tau} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$

Table 1: Table of Analogues Between Linear and Rotational Motion

1.2 Rotation of Rigid Bodies

- The distance between any two points in a rigid body remains constant.
- As a rigid body rotates about some fixed axis O, every point P in the body necessarily rotates with the same angular velocity and angular acceleration.

1.2.1 Rotational Kinetic Energy and Moment of Inertia

The total rotational energy is the sum of the rotational energies of each mass or molecule that makes up the rigid body:

$$\begin{aligned} \mathrm{KE_{rot}} &= \sum(\frac{1}{2}m_i v_i^2) \\ &= \sum(\frac{1}{2}m_i r_i^2)\left(\frac{v_i}{r_i}\right)^2 \\ &= \frac{1}{2}\left[\sum m_i r_i^2\right]\omega^2 \end{aligned} \tag{1}$$

where we have use the fact that ω is the same for every point in the rigid body to remove it from the sum. For rigid bodies, we cannot really sum over each molecule, so we split the body into *infinitesmal* pieces, each of mass dm at a distance r from the axis of rotation, and the sum becomes an integral:

$$\text{KE}_{\text{rot}} = \frac{1}{2} I \omega^2$$

where I is the moment of inertia of the body about the axis of rotation:

$$I = \int_{\text{body}} dm r^2$$

Shape	Moment of Inertia
Uniform hoop/cylinder about center	$I = MR^2$
Uniform Disc/Solid Cylinder about center	$I = \frac{1}{2}MR^2$
Solid Sphere about any axis through center	$I = \frac{2}{5}MR^2$
Hollow Sphere about any axis through center	$I = \frac{2}{3}MR^2$
Uniform Rod about axis through center	$I = \frac{1}{12}ML^2$
Uniform Rod about axis through end	$I = \frac{1}{3}ML^2$

Table 2: Table of Moments of Inertia of Various Rigid Bodies

Note

- The moment of inertia depends not only on the mass and size of the object, but also how the mass is distributed about the axis of rotation. The more mass there is further away from the rotation axis, the greater the moment of inertia.
- When applying the work-energy theorem (energy conservation) to rigid bodies, both translational kinetic energy and rotational kinetic energy must be taken into account.

1.2.2 Newton's Second Law for Rotating Rigid Bodies

The general motion of a rigid body can be completely described by the linear motion of its **center of mass** and its rotation about the center of mass.

Newton's Second Law therefore has two parts:

Linear Motion

$$\vec{F}_{\text{net}} = M\vec{a}_{\text{cm}} = \frac{d\vec{p}_{\text{tot}}}{dt}$$

where $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + ...$ is the net force on the object, M is the total mass, $\vec{a}_{\text{cm}}$ is th acceleration of the center of mass of the object, and $\vec{p}_{\text{tot}} = M\vec{v}_{\text{cm}}$ is its total linear momentum.

Rotational Motion

$$\vec{\tau}_{\text{net}} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$$

where $\tau_{\text{net}} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + ...$ is the net torque on the object, I is its moment of inertia about its center of mass, and $\vec{L}$ is its angular moment about the center of mass.

Note

- When applying Newton's laws to solve for the motion of a system of objects, one must apply the rotational version of the Second law to each extended object in the system that is free to rotate.

- In the absence of net external torque the total angular momentum of any system of objects (about any axis) is conserved.

1.3 Suggested Problems

10.15 *

10.24

10.25 *

10.33 *

10.39 *

10.43

11.11 *

11.25 *

11.33 *

11.41

11.54

Problems marked by * will be covered in tutorials.

2 GRAVITATION–Ch.14

2.1 Newton's Theory

In the 17th century, Sir Isaac Newton formulated his Law of Universal Gravitation to explain the observed motions of astronomical objects such as the moon and the planets. It can be summarized in words as:

"Every particle attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the distance between them"

2.1.1 Equation Form of Newton's Law of Gravitation

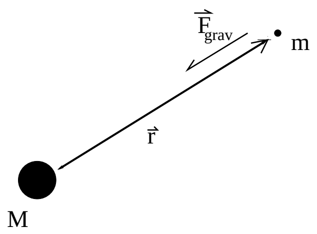

Figure 3: Gravitational force on m due to M

$$\vec{F}_{grav} = -\frac{GMm}{r^2}\hat{r} \tag{2}$$

$$G = 6.67 \times 10^{-11} Nm^2/kg^2 \tag{3}$$

2.1.2 Gravitational Field/Gravitational Acceleration

$$\vec{g} = \frac{\vec{F}_{grav}}{m} = -\frac{GM}{r^2}\hat{r} \tag{4}$$

NOTE: At the Earth's surface: $g = g_E = \frac{GM_E}{R_E^2} = 9.8m/s^2$

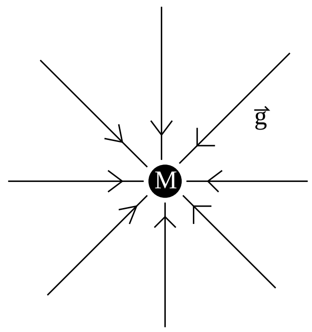

Figure 4: Acceleration field around a mass M

2.1.3 Gravitational Potential Energy

$$U = -\frac{GMm}{r} \tag{5}$$

Note:

a) $U = 0$ when $r = \infty$ by definition

b) Escape Velocity: give object enough kinetic energy to overcome gravitational potential

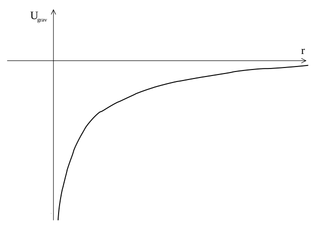

Figure 5: Gravitational Potential Energy

energy.

$$1/2mv_E^2 = \frac{GMm}{r} \rightarrow v_E = \sqrt{2GM/r} \tag{6}$$

2.2 Circular Orbits

Consider the motion of planets and satellites in circular orbits.

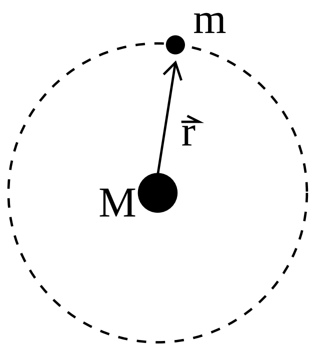

Figure 6: Circular Orbit

Gravity Provides Centripetal Force:

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \tag{7}$$

Total energy :

$$\begin{aligned} E &= 1/2mv^2 - \frac{GMm}{r} \\ &= \frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r} \\ &= = -\frac{1}{2}\frac{GMm}{r} \end{aligned}$$

NOTE: The above works only if the mass of the satelite/planet is very small compared to the mass of the object that it is orbitting. In general, both objects orbit their mutual center of mass,as shown below:

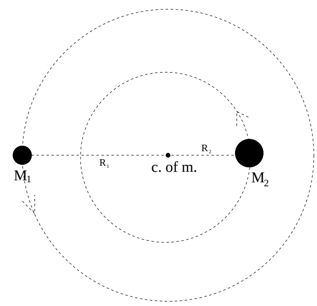

Figure 7: Circular Orbit for Near Equal Masses

2.3 Kepler's Laws:

2.3.1 1st Law: Planets Orbit in Elliptic Paths

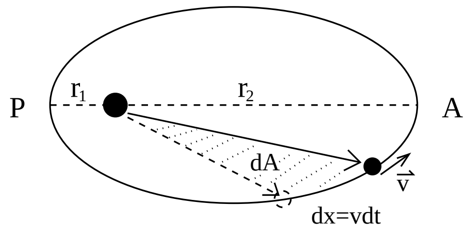

Figure 8: Elliptic Orbit

P = perigee(satellites)/perihelion(planets)

A = apogee(satellites)/aphelion(planets)

a = $\left(\frac{r_1 + r_2}{2}\right)$ = semi-major axis

2.3.2 2nd Law: Radius vector sweeps out equal areas in equal times.

Proof) From the rotational analogue of Newton's 2nd law, namely $\vec{\tau} = \frac{d\vec{L}}{dt}$ where $\vec{\tau} = \vec{r} \times \vec{F}$ (torque) and

$$\vec{L} = I\vec{\omega} = \vec{r} \times \vec{p} \tag{8}$$

we find that

$$L = mvr\sin\theta = \text{ constant} \tag{9}$$

which implies $dA = \frac{L}{2m}dt = \text{ constant } \times dt$ (see diagram above)

2.3.3 3rd Law: The square of the period is proportional to the cube of the semi-major axis.

$$T^2 = \frac{4\pi^2}{GM}a^3 \tag{10}$$

2.4 Einstein's Theory of Relativity

2.4.1 Problems with Newton's Law of Gravitation:

1. It gave the wrong prediction for the precession of Mercury's orbit

2. It didn't explain why the gravitational force on an object was proportional to its *inertial* mass (i.e. why objects fell with an acceleration independent of their mass and composition).

3. It was inconsistent with Einstein's Theory of Special Relativity: if an instantaneous force of attraction existed between two distant objects, information about the location of one is instantaneously transmitted to the other (i.e faster than the speed of light)

2.4.2 Main Features of General Relativity

1. Space has structure: it can be curved. A two dimensional representation of what the curved space around the Sun might look like is given in Fig.9.

2. Matter causes space to curve.

3. Small objects travel along the straightest possible line in curved space (space-time): i.e. space tells matter how to move

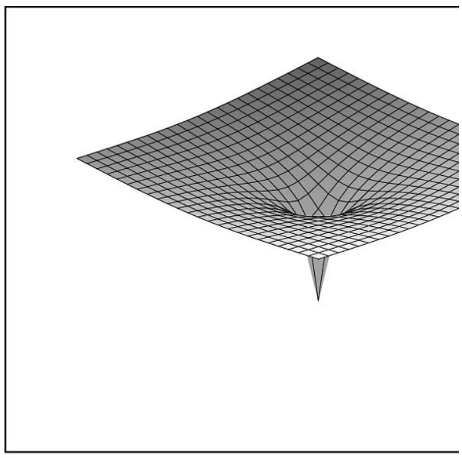

Figure 9: Curved space around a massive object

2.4.3 Black Holes

1. **Definition:** region of space so densely packed with matter that nothing, not even light, can escape (i.e. escape velocity greater than speed of light)

2. Black holes form by gravitational collapse of stars that have burned up all thermonuclear fuel or by collapse of stars at center of galaxy.

3. surface that separated the region of no return from the rest of the universe is called the *event horizon* of the black hole

4. Black holes that weigh one solar mass have a radius of about 5 km.

5. At the center of a black hole is a *singularity*, where all the mass of the black hole is concentrated and the known laws of physics break down.

6. Inside the event horizon time and space exchange roles; once you fall below the event horizon you can no more avoid falling to the singularity at the center than you can avoid moving from 2:00 o'clock to 3:00 o'clock when you are outside.

2.5 Suggested Problems

14.2*

14.3

14.15*

14.19

14.24

14.28

14.30*

14.33*

14.35*

14.38*

14.64

3 SIMPLE HARMONIC MOTION – Ch. 13

Any system that is displaced slightly from a stable equilibrium position will experience a restoring force and oscillate about that equilibrium position. If the initial displacement is small enough, the restoring force is proportional to the displacement and the system undergoes simple harmonic motion.

3.1 General Properties

Defining relation:

$$F = -kx \Leftrightarrow \frac{d^2x}{dt^2} = -\omega^2 x \tag{11}$$

$$\omega = \sqrt{k/m} \tag{12}$$

Solution:

$$x = A\cos(\omega t + \phi) = a\cos(\omega t) + b\sin(\omega t) \tag{13}$$

$$v = -\omega A \sin(\omega t + \phi) \tag{14}$$

Amplitude:

$$|x_{max}| = A \tag{15}$$

Period:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{m/k} \tag{16}$$

Maximum Speed:

$$|v_{max}| = \omega A = \sqrt{k/m}A \tag{17}$$

Mechanical energy:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \tag{18}$$

Initial Conditions:

The amplitude and phase can be determined from the initial position, x_a, and velocity, v_a, at any initial time $t = t_a$:

$$A = \sqrt{x_a^2 + \frac{m}{k} v_a^2} = \sqrt{x_a^2 + (v_a/\omega)^2} \tag{19}$$

$$\phi = \cos^{-1}(x_a/A) - \omega t_a = \tan^{-1}\left(\frac{-v_a}{\omega x_a}\right) - \omega t_a \tag{20}$$

NOTE: Both $\cos^{-1}$ and $\tan^{-1}$ are ambiguous: you must check that you have the correct phase, by making sure that it yields the correct x_a,v_a at time t_a.

Graph of solution: Need to know the amplitude, A, period, T and the phase ϕ, or equivalently, the time $t_0 = -\phi/\omega$ that the oscillator goes through maximum displacement $x = A$.

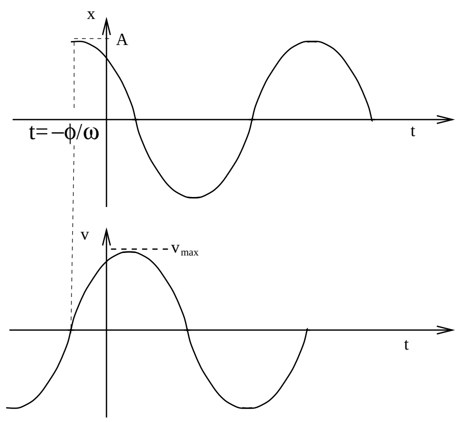

Figure 10: Position-time and velocity-time graphs for the SHO

3.2 The Physical Pendulum

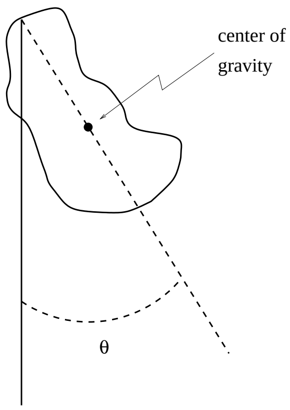

Figure 11: The Physical Pendulum

$$\frac{d^2\theta}{dt^2} = -\left(\frac{mgL}{I}\right)\sin\theta \approx -\frac{mgL}{I}\theta \tag{21}$$

where I is **the moment of inertia** of the pendulum, and the last expression is approximately true for small angular displacements θ (rads) $<< 1$ **Angular Frequency:** $\omega = \sqrt{mGL/I}$ **Period:** $T = 2\pi\frac{I}{mGL}$

3.2.1 Simple Pendulum:

$I = mL^2$

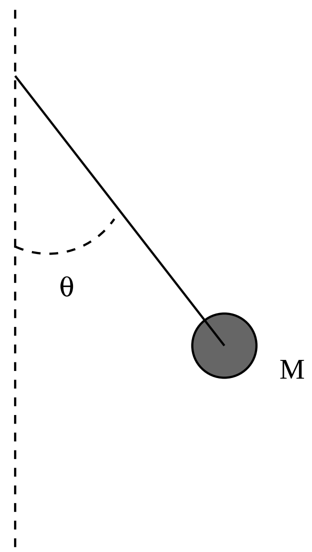

Figure 12: Simple Pendulum

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \qquad T = 2\pi\sqrt{\frac{L}{g}} \tag{22}$$

3.2.2 Metre Stick:

$I = \frac{1}{3}ml^2$

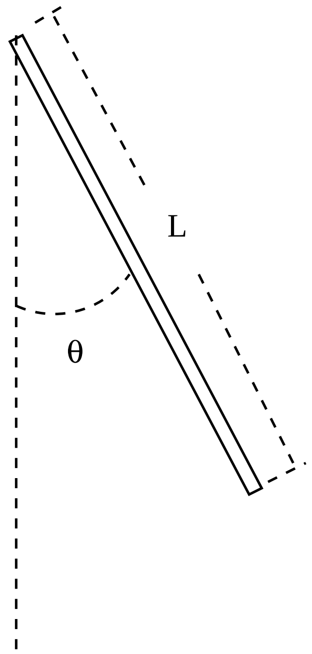

Figure 13: The Meter Stick

$$\frac{1}{3}ml^2\frac{d^2\theta}{dt^2} = -mg\frac{l}{2}\sin\theta \tag{23}$$

$$\rightarrow \frac{d^2\theta}{dt^2} \sim -\frac{3}{2}g/l\ \theta \tag{24}$$

3.3 Suggested Problems

13.5

13.7

13.9 *

13.17

13.20

13.21

13.22

13.25 *

13.26

13.29

13.59 *

4 WAVES – Ch. 16–18.

4.1 General Properties

4.1.1 Types of Waves:

1. Mechanical

2. Electromagnetic/gravitational

3. Matter

4.1.2 Types of Wave Propagation:

1. Longitudinal (*eg. spring, sound waves*)

2. Transverse (*eg String, electromagnetic*)

3. Mixed (*eg. Water*)

4.1.3 Wave Pulse:

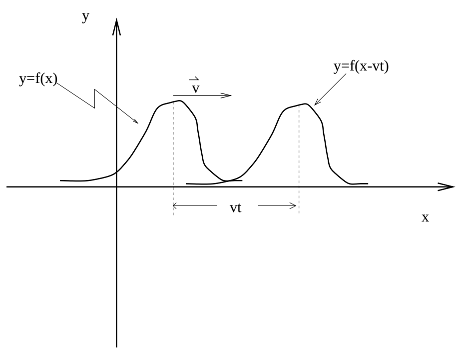

Figure 14: Moving wave pulse

$y = f(x - |v|t)$; moves along +ve x-axis, speed $|v|$

$y = f(x + |v|t)$; moves along -ve x-axis, speed $|v|$

4.2 Sinusoidal Waves:

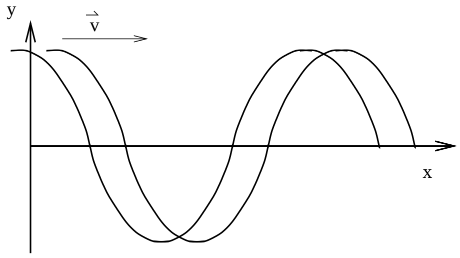

Figure 15: Moving sine wave

$$\begin{aligned} y &= A\sin(kx-\omega t-\phi) \\ &= A\sin\left(\frac{2\pi}{\lambda}(x-vt)-\phi\right) \end{aligned}$$

$$f=\frac{|v|}{\lambda}=\frac{\omega}{2\pi};\quad k=\frac{2\pi}{\lambda};\quad vt=\frac{\omega}{k} \tag{25}$$

Transverse velocity:

$$v^{trans}=\left.\frac{\partial y}{\partial t}\right|_x=-\omega A\cos(kx-\omega t-\phi) \tag{26}$$

Wave equation:

$$\frac{\partial^2 y}{\partial t^2}-v^2\frac{\partial^2 y}{\partial x^2}=0 \tag{27}$$

Speed of transverse wave on string:

$$v=\sqrt{F/\mu}\qquad \text{(from Newton's 2nd Law)} \tag{28}$$

Energy transmitted by sinusoidal wave:

$$\frac{dE}{dt}=\frac{1}{2}\mu\omega^2A^2v\quad \text{(watts)} \tag{29}$$

4.3 Reflection and Transmission:

- incident from less dense to more dense: reflected part inverted ($180°$ phase change) transmitted part slows down ($v_A > v_B$)
- incident from more dense to less dense: reflected part no phase change transmitted part speeds up ($v_A > v_B$)

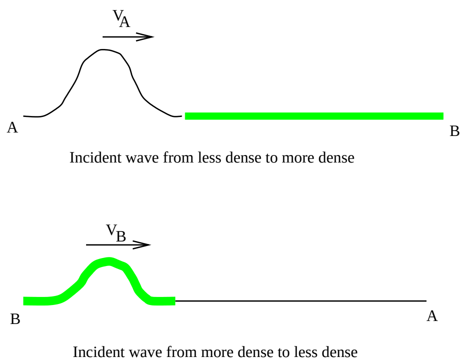

Figure 16: Reflection and transmission on a string

4.4 Sound Waves:

- longitudinal compression waves
- audible frequency range 20 Hz – 20,000 Hz
- speed of sound in air (at 20°C) 343 m/s
- Doppler effect: $f' = f\dfrac{(v \pm v_o)}{(v \mp v_s)}$

4.5 Superposition of Waves:

4.5.1 General:

$$y = y_1 + y_2$$

4.5.2 Constructive/destructive interference:

$$\begin{aligned} y &= A\sin(kx-\omega t) + A\sin(kx-\omega t-\phi) \\ &= 2A\cos(\phi/2)\sin(kx-\omega t-\phi/2) \end{aligned}$$

– constructive when $\phi = 0, \pm 2\pi, \pm 4\pi,$ (in phase)

– destructive when $\phi = \pm\pi, \pm 3\pi, \pm 5\pi$ (180° out of phase)

4.5.3 Two Sources, In Phase, Same Frequency: but different paths:

Phase Shift:

$$\begin{aligned} \phi = \left|\frac{r_1 - r_2}{\lambda}\right| 2\pi \quad \rightarrow \quad & |r_1 - r_2| = n\lambda \qquad \text{constructive} \\ & |r_1 - r_2| = (n + \frac{1}{2})\lambda \qquad \text{destructive} \end{aligned} \tag{30}$$

4.5.4 Standing Waves:

$$y = A\sin(kx-\omega t) + A\sin(kx+\omega t) = 2A\sin(kx)\cos(\omega t)$$

Nodes: $x = \frac{n}{2}\lambda$

4.5.5 Modes of a String Fixed at Both Ends:

$\lambda_n = \frac{2}{n}L$

$f_n = \frac{v}{\lambda_n} = \sqrt{\frac{F}{\mu}}\frac{n}{2L}$

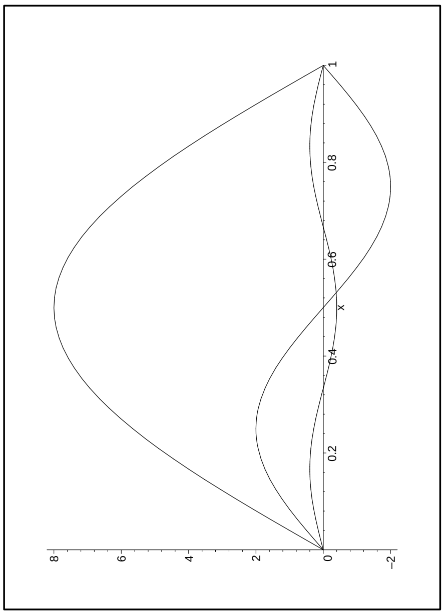

Figure 17: First 3 Fundamental Modes of a String

4.6 Suggested Problems–Chapters 16-18

16.36*

16.51*

17.35*

17.39*

18.1*

18.8*

18.13*

18.22*

5 WAVE PROPERTIES OF LIGHT – Ch. 35–37

5.1 General

- Qualitative Properties of Electromagnetic Spectrum:
  gamma rays; uv; visible; IR; micro; tv/radio
- speed of light in vacuum: $c = 3 \times 10^8$m/s
- frequency: $f = c/\lambda$

5.2 Refraction

5.2.1 Index of refraction:

$n = c/v > 1$

5.2.2 Snell's Law:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \tag{31}$$

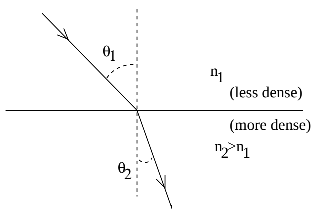

Figure 18: Refraction

5.2.3 Total internal reflection:

$$\sin\theta_C = \frac{n_1}{n_2} < 1 \tag{32}$$

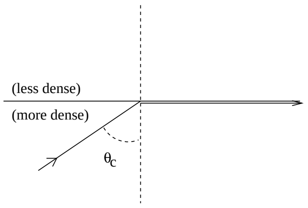

Figure 19: Total Internal Reflection

5.2.4 Dispersion:

Index of refraction decreases with increasing wavelength, so that blue light has higher index of refraction and bends more than red light.

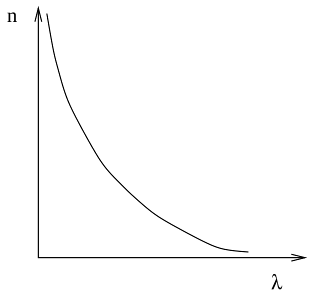

Figure 20: Change of index of refracton with wavelength

5.2.5 Rainbows:

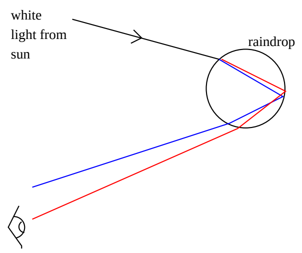

Figure 21: Refraction, Diffraction and Internal Reflection of Light Inside a Raindrop

5.3 Interference

$r_1 - r_2) = m\lambda$ constructive

$(r_1 - r_2) = (m + 1/2)\lambda$ destructive

5.3.1 Huygen's Principle

5.3.2 Young's Double Slit Experiment

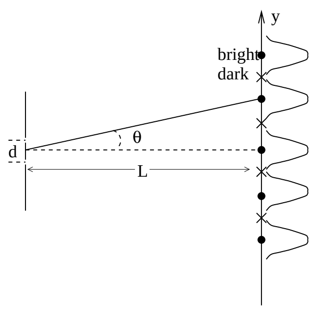

Figure 22: Young's Double Slit Experiment

$\sin\theta^{Bright} = \frac{y^{Bright}}{L} = m\frac{\lambda}{d}$

$\sin\theta^{Dark} = \frac{y^{Dark}}{L} = (m + 1/2)\frac{\lambda}{d}$

5.3.3 Thin Films:

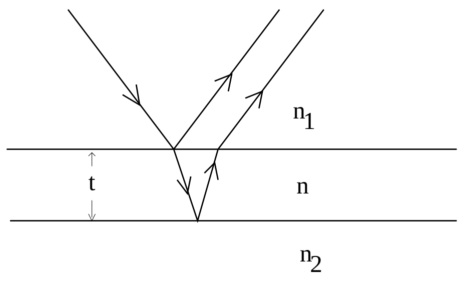

Figure 23: Reflection by Thin Film

For $n > n_1$ and $n > n_2$:
destructrive interference occurs when $2t = m\frac{\lambda}{n}$
constructive interference occurs when $2t = (m + 1/2)\frac{\lambda}{n}$
NOTE: 180° phase change on reflection of more dense material

5.3.4 Single Slit Diffraction:

$$\sin\theta^{Dark} \simeq \frac{y^{Dark}}{L} = m\frac{\lambda}{a} \tag{33}$$

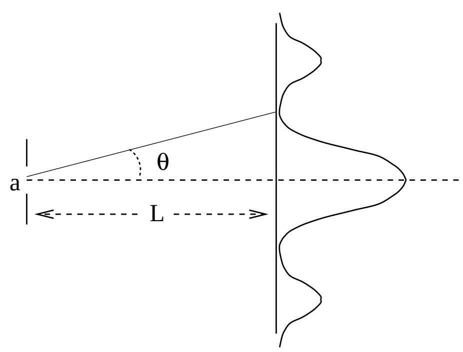

Figure 24: Single Slit Diffraction

5.4 Suggested Problems

CHAPTER 35.

35.6*

35.10

35.18

35.28*

35.39*

CHAPTERS 37 & 38.

37.1

37.2*

37.4

37.30

37.31

37.32

37.59*

38.2*

6 QUANTUM MECHANICS – Ch. 40, 41

6.1 BlackBody Radiation/UV Catastrophe

Planck's Postulates

1.) $E = nhf$ (energy of molecules quantized)

2.) $E_\gamma = hf$ (energy of light comes in "lumps")

$$h = 6.63 \times 10^{-34} \mathrm{J} \cdot \mathrm{S}$$

6.2 Photoelectric Effect

$KE_{max} = hf - \phi \qquad f_C = \phi/h$

$(1eV = 1.6 \times 10^{-19} \mathrm{J})$

Einstein's Explanation:

– light made up of a stream of particles with energy $E_\gamma = hf$

6.3 Compton Effect:

Scatter x-rays off electrons in carbon

$$E_\gamma = hf = \frac{hc}{\lambda} \to \lambda' - \lambda = \frac{h}{m_e C}(1 - \cos\theta)$$

$$p_\gamma = \frac{E_\gamma}{C} = \frac{h}{\lambda}$$

6.4 de Broglie Wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

6.5 Wave Mechanics

Particles described by wavefunction $\psi(x)$; probability, P, of finding particle between x and $x + dx$ is

$$P = |\psi(x)|^2 dx \tag{34}$$

6.6 Consequences of Wave Mechanics:

1. Electron Interference Pattern

2. Uncertainty Principle

$$\triangle x \triangle p \geq \frac{h}{2\pi} \tag{35}$$

3. Particle in a Box:

$$\psi_n(x) = A \sin\left(\frac{n\pi x}{L}\right) \tag{36}$$

   (fundamental modes) $\quad n = 1, 2, 3, ...$

$$P_n = \frac{h}{\lambda_n} = \frac{nh}{2L} \tag{37}$$

$$E_n = \frac{P_n^2}{2m} = \frac{n^2 h^2}{8mL^2} \tag{38}$$

4. Hydrogen Atom:

$$mv_n r_n = nh \tag{39}$$

$$r_n = a_o n^2 \tag{40}$$

$$E_n = -\frac{ke^2}{2a_o}\frac{1}{n^2} \tag{41}$$

6.7 Suggested Problems

40.1*

40.9*

40.16*

40.25*

40.54*

41.1*

41.5*

41.10*